Social Badge System Analysis

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***Abstract*—To incentivize users' participations, online social networks often provide users with various rewards for their contributions to the sites. Attracted by the rewards, users will spend more time using the network services. Specifically, in this paper, we will mainly focus on "badges reward systems". Badges are small icons attached to users' homepages and profiles denoting their achievements. People like to accumulate badge for various reasons, which are modeled as the "*badge values*" in this paper. Meanwhile, to get badges, people also need to exert efforts to finish the required tasks, which will lead to certain "*costs*" as well. To understand users' badge achievement activities better, we will study an existing badge system launched in a real-world online social network, Foursquare, in this paper. A longer version of this paper is available at [14].**

***Index Terms*—Badge System, Social Network, Data Mining**

I. Badge System Introduction

Online social networks, e.g., Facebook, Twitter and Foursquare, have achieved remarkable success in recent years. These social networks are mostly driven by user-generated content, e.g., posts, photos and location checkins. To incentivize users' participations and steer their online activities, many social networks start to offer users various kinds of rewards for their contributions to the networks. In this paper, we will mainly focus on "badge reward systems" but the proposed models can be applied to other reward systems as well.

Badge systems have been adopted by a wide range of social networks: (1) Foursquare[1], a famous location-based social network (LBSN), is distributing different badges to users for their geo-location checkins; (2) Weibo[2], a social media in China, launches a badge system to give users badges for writing posts and replies; (3) Stack Overflow[3], a popular question and answer (Q&A) site, adopts a system where users can get badges by answering questions in the site; and (4) In Khan Academy[4], a popular massive open online course (MOOC) site, users are awarded badges for watching course videos and answering questions. For instance, in Figure 1, we show the top 10 badges achieved by the most users in Foursquare, which cover very diverse user social activities in the offline world.

[1]https://foursquare.com
[2]http://www.weibo.com
[3]http://stackoverflow.com
[4]https://www.khanacademy.org

Fig. 1. Top 10 badges achieved by most Foursquare users

A. Real World Badge System Dataset

To study the users' badge achievement activities more clearly, we have conducted extensive analyses on a real-world badge system launched in Foursquare. The badge system dataset was crawled from Foursquare during the April of 2014, whose statistical information is available in Table I.

TABLE I
Properties of the Badge System Dataset

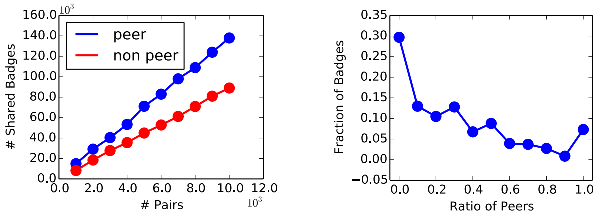
	property	number
nodes	user	4,240
	badge	1,431
links	follow	81,291
	achieved	176,301

We collected $4,240$ Foursquare users together with all the $1,430$ categories of badges achieved by them, where each category of badges can involve different badge levels. These users are crawled with BFS search from several random seed users via the social connections in Foursquare, whose number is $81,291$ among the crawled users. To denote that a user has achieved certain badges, we add *achieve* links between users and badges, whose total number is $176,301$ in the crawled dataset. On average, each user has achieved 42 badges in Foursquare.

B. User Badge Achievement Motivations

Users in online social networks like to accumulate badges for various motivations. By studying the crawled Foursquare badge dataset, we have several important observations about users' badge achievement activities.

Observation 1: Users who are friends are more likely to get the common badges together. We randomly sample a certain



(a) shared badge distribution (b) peer pressure distribution

Fig. 2. Statistical information: (a) #shared badges between randomly sampled user pairs; (b) fraction of badges obtained at different friend ratios.

number of user pairs who are (1) friends (i.e., connected by social links) and (2) not friends from Foursquare, and count the number of common badges achieved by these user pairs on the same day. The results are given in Figure 2(a), where the x axis is the number of randomly sampled user pairs and the y axis denotes the number of shared badges between these sampled pairs. From Figure 2(a), we can observe that online badge achievement in social networks is correlated with social connections among users and friends are more likely to get common badges at the same time.

Observation 2: Users like to obtain badges never achieved by their friends. Besides getting common badges with close friends, Foursquare users also like to get new badges that have never been obtained by their peers before. As shown in Figure 2(b), for each badge b_j, obtained by user u_i, we get the timestamp when u_i get b_j and the ratio of u_i's friends who obtain b_j before u_i. The distribution of the percentage of badges obtained at different ratios is given in Figure 2(b), from which we can observe that a large proportion of badges are obtained at small ratios (e.g. 0), which denotes none of u_i's friends have achieved the badge before u_i.

Observation 3: Users will follow their peers when most of them have obtained a certain badge. Still in Figure 2(b), when the ratio is close to 1.0 (i.e., all the peers have the badge), the fraction of badge obtained will increase to 0.1, which represents that about 10% of the badges are obtained by users when all his friends have achieved the badge. In other words, Foursquare users will also follow their peers to extricate themselves from the backward positions.

Therefore, from the analysis results, we observe that users' badge achievement activities in online social networks are highly related to those of their peers. Formally, these observed peers' influences on users' badge achievement in online social networks are modeled as the *peer pressure value* of badges in this paper.

Observation 4: Users are keen on getting badges to their own interests. Besides the influences from the peers, users' own personal interests also play an important role in steering their badge achievements activities. In Table II, we extract top 10 popular badges achieved by the most users in Foursquare and each of these badges has 10 different levels. Numbers of users who have obtained certain levels of each kind of badge are provided, whose icons are shown in Figure 1. Generally, higher-level badges require more efforts from the users, but from Table II, we observe there are still a large number of users being willing to devote such high efforts to get these badges. For example, among the $2,468$ users who achieved the "Fresh Brew" badge of level 1, 22.5% of them will continue to get the badge of level 5.

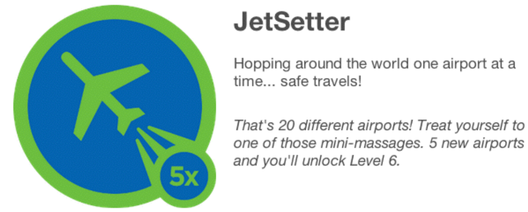


Fig. 3. Tasks needed to unlock "JetSetter" badges.

As a result, these obtained badges can reveal users' personal interests, especially higher-level badges. To model the attractions of these badges on users, we introduce the concept of *personal interest value* about the badges in this paper.

Observation 5: Users in the network are enthusiastic in earning badges. From the results in Table II, users generally obtain the the same badges of different levels sequentially from the lower level to higher levels. In addition, from the crawled dataset, many other interesting relative badge achievement sequential patterns can be observed, which is not due to the *peer pressure value* nor the *personal interest value* of the badges.

Therefore, there exists a global influence from the network affecting all the individuals' badge achievement activities in the network. Such kind of effects of badges on users is modeled with the concept of *network steering value* about badges in this paper.

Besides the above badge values, users can also get other benefits in badge achievement from the network, e.g., enjoy the social network services. With the same modeling method, such benefits can be handled by either incorporating them into the above badge value categories or introducing a new value category. To simplify the problem setting, we will only consider these above badge values in this paper.

C. User Badge Achievement Costs and Utility

To get badges, users in online social networks are required to finish certain tasks, which can be (1) finishing a certain number of checkins at required locations in Foursquare, (2) answering a number of questions proposed by other users in Stack Overflow, and (3) publishing the required numbers of posts in Weibo. These tasks of higher-level badges are usually very hard. For instance, as shown in Figure 3, by checking at 20 different airports, users can get the "JetSetter" badges from level 1 to level 5. However, to unlock the "JetSetter" badge of level 6, users need to check in at another 5 new airports. Obviously, these tasks can yield some costs, which can be time, money or knowledge spent on the tasks.

By taking the *values* and *costs* of badges into consideration simultaneously, the payoff of achieving certain badges is defined as "(value - cost)", i.e., the utility of badges for users. When the value of badges can exceed the cost, users may try to get the badge; otherwise, they will not devote their efforts as they can get no payoffs from these badges. Costs of obtaining badges are fixed but the value of badges can be influenced by other users' badge achievement activities. Each user in online social network is assumed to be "selfish" and wants to maximize his payoff (i.e., the utility) and the badge

TABLE II
NUMBER OF USERS ACHIEVING TOP 10 BADGES

badge name	obtain it by checking-in at	# users achieving badges of different levels 1	2	3	4	5	6	7	8	9	10	total number
Fresh Brew	Coffee Shops	2468	1914	1235	817	555	374	255	144	78	38	7878
Mall Rat	Shopping Malls	2545	1907	1076	624	366	224	130	81	46	29	7028
JetSetter	Airport Terminals	2357	1703	972	564	339	210	147	102	63	11	6468
Hot Tamale	Mexican Restaurants	2305	1733	989	583	336	191	105	58	37	18	6355
Great Outdoors	Parks and Outdoors	2119	1535	801	468	295	200	132	95	53	30	5728
Pizzaiolo	Pizza Restaurants	2192	1450	605	267	116	62	26	16	8	4	4746
Swimmies	Lake/Pond/Beach	1888	1214	538	281	159	107	74	47	36	17	4361
Bento	Sushi Restaurants	1741	1121	459	209	104	63	34	21	14	8	3774
Zoetrope	Movie Theaters	1985	1106	309	103	34	16	12	6	5	4	3580
Flame Broiled	Burger Restaurants	1944	1044	337	105	40	13	6	3	1	1	3494

achieving activities will form a *game* with other users in the network.

D. Road Map

The following parts of this paper are organized as follows. We will first introduce the definitions of many important concepts, and then talk about the various *value functions* of badges for users. After that, based on the badge value and cost concepts, we will introduce the users' *utility function*, and study the game among users in badge achievement. Finally, we will talk about the related works and conclude this paper.

II. TERMINOLOGY DEFINITION

Users in social networks can be gifted in different areas and they can finish the tasks required to get badges corresponding to their gifts effortlessly. For instance, in Foursquare, sports enthusiasts can get *Gym Rat* badges easily as they do sports in gyms regularly, while travel lovers can obtain *JetSetter* or *Trainspotter* badges by checking in at train stations and airports frequently. However, for users who want to get badges of areas that they are not good at, it would be very difficult to finish the required tasks. For example, a sports enthusiast may need to spend lots of time and money to get the *JetSetter* or *Trainspotter* badges by travelling. Similarly, gourmets who seldom do sports may suffer a lot to get *Gym Rat* badges by visiting gyms. Let $\mathcal{U} = \{u_1, u_2, \cdots, u_n\}$ and $\mathcal{B} = \{b_1, b_2, \cdots, b_m\}$ be the sets of n users and m badges respectively in the network. To depict such phenomena, we formally define the concepts of *ability*, *effort* and *contribution* of users in $\mathcal{U}$ as well as *contribution threshold* of badges in $\mathcal{B}$ as follows.

Definition 1 (Ability): User u_i's talents or advantages in fields corresponding to badges in $\mathcal{B}$ can be represented as the *ability* vector $\mathbf{a}_i = [a_{i,1}, a_{i,2}, \cdots, a_{i,m}]$, where $a_{i,j} \geq 0$ denotes u_i's *ability* in the field of badge b_j.

All people are assumed to be created equally talented, but they can be talented at different aspects. For simplicity, we assume the total abilities of different users are equal, i.e., $|\mathbf{a}_i|_1 = |\mathbf{a}_j|_1$, for $\forall u_i, u_j \in \mathcal{U}$. Besides talents, to make achievements in certain areas, every people need to devote their efforts and passion, which can be either money, time, energy or knowledge.

Definition 2 (Unit Time Effort): Vector $\mathbf{e}_i = [e_{i,1}, e_{i,2}, \cdots, e_{i,m}]$ denotes user u_i's efforts devoted to the field corresponding badges in $\mathcal{B}$ in unit time, where $e_{i,j} \geq 0$ represents u_i's *effort* devoted in the area of badge b_j.

Users' *unit time effort* can vary with time and can be represented as a function on time, e.g., $e_{i,j}(t)$. The total amount of *unit time effort* in different areas of all users are assumed to be equal, i.e., $|\mathbf{e}_i|_1 = |\mathbf{e}_j|_1$, for $\forall u_i, u_j \in \mathcal{U}$. Meanwhile, the more time people devote to certain area, the more *cumulative efforts* he will devote to the area.

Definition 3 (Cumulative Effort): Term $\hat{e}_{i,j} = \int_{\underline{t}}^{\bar{t}} e_{i,j}(t)\, dt$ is defined as the *cumulative effort* that user u_i devotes to badge b_j during time period $[\underline{t}, \bar{t}]$. For users, *cumulative effort* is more meaningful as they only care about the total amount of effective efforts devoted to the system. Vector $\hat{\mathbf{e}}_i = [\hat{e}_{i,1}, \hat{e}_{i,2}, \cdots, \hat{e}_{i,m}]$ is defined as the *cumulative efforts* that user u_i pays to the network.

In this paper, active users are assumed to have more *cumulative efforts*. The achievements people obtain depend on not only their ability in a certain area but also the efforts the devoted to the area, which can be formally defined as their *contributions* to the network.

Definition 4 (User Contribution): The effectiveness of users' cumulative efforts devoted to a social network is formally defined as their *contributions*. Vector $\mathbf{c}_i = [c_{i,1}, c_{i,2}, \cdots, c_{i,m}]$ is defined to be user u_i's contributions to the whole system, where $c_{i,j}$ is the contribution of user u_i devoted to the network in getting badge b_j during $[\underline{t}, \bar{t}]$:

$$c_{i,j} = \int_{\underline{t}}^{\bar{t}} a_{i,j} e_{i,j}(t)\, dt = a_{i,j} \int_{\underline{t}}^{\bar{t}} e_{i,j}(t)\, dt = a_{i,j} \hat{e}_{i,j}.$$

As a result, the more effort people devote to areas they are gifted at, the more remarkable achievements they can get in the areas. In social networks, whether a user can receive a badge depends on not only the contributions he/she make but also the contribution *threshold* of the badge.

Definition 5 (Badge Threshold): A badge's *threshold* denotes the minimum required *contributions* for users to get the badge. For badges in $\mathcal{B}$, their *threshold* can be represented as $\theta = [\theta_1, \theta_2, \cdots, \theta_m]$.

For a given user u_i, if his/her contribution to badge b_j, i.e., $c_{i,j}$, is greater than b_j's threshold θ_j, then u_i will get b_j, which can be represented with the *badge indicator function*: $I(c_{i,j} \geq \theta_j) = \begin{cases} 1, & \text{if } c_{i,j} \geq \theta_j, \\ 0, & \text{otherwise.} \end{cases}$. Furthermore, the badges that user

u_i have received can be represented as the *badge indicator vector* $\mathbf{I}_i = [I(c_{i,1} \geq \theta_1), I(c_{i,2} \geq \theta_2), \cdots, I(c_{i,m} \geq \theta_m)]$. Before the system starts to operate and players begin to invest their efforts, the badge system designer needs to specify the badge system settings in advance.

III. Badge Value Functions

The motivation of users being willing to devote efforts to get badges in online social networks is because these badges are attracting to them, and the attraction of badges is modeled as the badge value in this paper. Depending on the specific scenarios, the value of badges for users can be quite different. According to the observations in the previous section, the effects of badges on users' badge achievement activities can be divided into three different categories, which will be described in this section in detail.

A. Peer Pressure Value Function

In our daily life, on the one hand, people want to be different from the public, while, on the other hand, they may also want to follow the mainstream as well. We have similar observations about users badge achieving activities in online social networks. Users in online social networks want to be the first to win certain badges in their communities, which can show their uniqueness and make them stand out from his/her peers. Meanwhile, if most of the peers have obtained a certain badge, users will follow their friends to get the badge to extract themselves from the backward position.

1) Peer Pressure Value Function Definition: To depict the effectiveness of badges to make users be either more superior to his peers or closer to other leading peers, we formally define the *peer pressure badge value* in this part.

Definition 6 (Peer Pressure Value): The *peer pressure value* of badge b_j for a user u_i is defined as a function about the ratio of u_i's peers who have obtained badge b_j already. Let $\Gamma(u_i)$ be the neighbor set of user $u_i \in \mathcal{U}$, in which users who have achieved badge b_j before u_i can be represented as $\Psi(u_i, b_j) = \{u_m | (u_m \in \Gamma(u_i)) \wedge (\mathbf{I}_m(j) = 1)\}$. The *peer pressure value* function of badge b_j for user u_i can be represented as function

$$v^{pp}(u_i, b_j|\Gamma(u_i)) = f(\frac{|\Psi(u_i, b_j)|}{|\Gamma(u_i)|}), \Psi(u_i, b_j) \subset \Gamma(u_i).$$

The concrete representation of the *peer pressure value* functions can be quite diverse depending on the selected function $f(\cdot)$. In this paper, we try 4 different functions, which include:

- *linear peer pressure value function* $v_l^{pp}(u_i, b_j|\Gamma(u_i)) = a\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right) + b$;
- *quadratic peer pressure value function* $v_p^{pp}(u_i, b_j|\Gamma(u_i)) = a\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right)^2 + b\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right) + c$;
- *cubic peer pressure value function* $v_c^{pp}(u_i, b_j|\Gamma(u_i)) = a\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right)^3 + b\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right)^2 + c\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right) + d$;
- *exponential pressure value function* $v_e^{pp}(u_i, b_j|\Gamma(u_i)) = a \times e^{-b\left(\frac{|\Psi(u_i,b_j)|}{|\Gamma(u_i)|}\right)} + c$;

where a, b, c and d are the coefficients in the functions. These parameters can be learnt by fitting these function to the historical data, whose results are shown in Figures 4(a)-4(d) respectively.

2) Peer Pressure Value Function Evaluation: The higher *peer pressure value* a badge has, the more likely a user will try to obtain it. To test the effectiveness of the above introduced *peer pressure value functions*, we conduct an experiments on the Foursquare badge system dataset introduced in the introduction section.

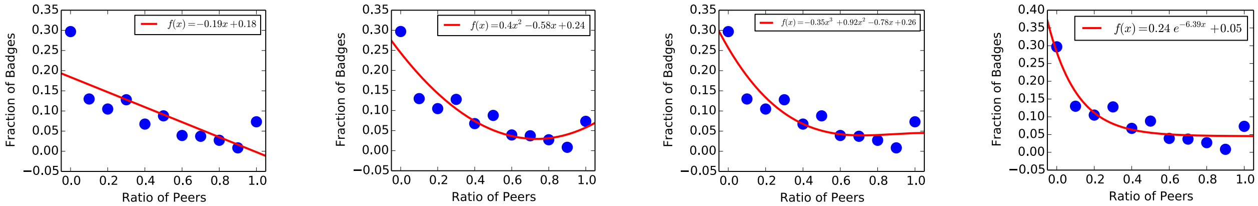
Experiment Settings

In the experiment, badges achieved by less than 100 users are removed and the remaining badges achieved by users are organized in a sequence of (user, badge) pairs according to their achieving timestamps. These (user, badge) pairs are divided into two subsequences according to their relative timestamps order: the training set and testing set, the proportion of whose sizes is $9:1$. In addition, a set of non-existing (user, badge) pairs which is of the same size as the positive test set are randomly sampled from the network as the negative test set, which together with the positive test set are used to form the final testing set. Pairs in the training set are regarded as the historical data, based on which we calculate the values of pairs in the testing set and output them as the confidence scores of these pairs.

The evaluation metrics applied in the experiment is AUC and Precision@100. In statistics, a receiver operating characteristic (ROC), or ROC curve, is a graphical plot that illustrates the performance of a binary classifier system as its discrimination threshold is varied. The curve is created by plotting the true positive rate (TPR) against the false positive rate (FPR) at various threshold settings. The area under the ROC curve is usually quantified as the AUC score. When using normalized units, the area under the curve (i.e., AUC) is equal to the probability that a classifier will rank a randomly chosen positive instance higher than a randomly chosen negative one (assuming "positive" ranks higher than "negative"). Generally speaking, larger AUC score corresponds to better performance of the prediction model. Meanwhile, Precision@100 measures the ratio of instances correctly predicted among those of the top 100 prediction scores.

Experiment Results

We learn the coefficients of different value functions with the training set and apply the learnt function to calculate the *peer pressure values* of pairs in the testing set. The results are available in Figure 5. From the results, we observe that AUC achieved by the *quadratic peer pressure function* is 0.65, which is slightly better than other value functions, and the AUC scores obtained by the *linear*, *cubic* and *exponential peer pressure functions* are 0.58, 0.63, and 0.62 respectively. Similar results can be observed when the evaluation metric is Precision@100. The *quadratic peer pressure function* can achieve a Precision@100 score of 0.44, which is higher than all the other *peer pressure functions*. Here, quadratic function can outperform cubic and exponential functions can because of the reason that cubic function may suffer from the overfitting problems a lot. Next, we will use the *quadratic peer pressure*



(a) linear value function (b) quadratic value function (c) cubic value function (d) exp. value function

Fig. 4. Estimated value functions by fitting the data.

function as the only *peer pressure function*, which will be compared with other value functions in Figure 6.

B. Personal Interest Value Function

Users generally have their own personal interests, which can steer their social activities in online social networks. Users' personal interests can be revealed from the badges obtained in the past. For example, for a given user u_i who has already achieved the "Gym Rat" badges of levels from 1 to 4, it can show that u_i can like doing sports a lot and "Gym Rat" of level 5 can be of great value to him. Viewed in this way, the value of badges can be evaluated with the badges that users obtained in the past.

Definition 7 (Personal Interest Value): For a given user u_i and the set of badges obtained by u_i in the past, i.e., $\mathcal{H}$, the *personal interest value* of badge b_j for user u_i is defined to be

$$v^{pi}(u_i, b_j|\mathcal{H}) = \frac{\sum_{b_k \in \mathcal{H}} s(b_j, b_k) v^{pi}(u_i, b_k)}{|\mathcal{H}|},$$

where $s(b_j, b_k)$ denotes the similarity score between badge b_j and b_k and $v^{pi}(u_i, b_k)$ represents the *personal interest value* of badge b_k for user u_i.

For badge $b_k \in \mathcal{H}$ that u_i has obtained in the past, we define the *personal interest value* of u_i to badge b_j as 1.0 (i.e., $v^{pi}(u_i, b_k) = 1.0$, for $\forall b_k \in \mathcal{H}$). The similarity score between any two badges, e.g., b_j and b_k, is defined as the *Jaccard's Coefficient* score [13] of user sets who have achieved b_j and b_k (i.e., $\Gamma(b_j)$ and $\Gamma(b_k)$) respectively in the network: $s(b_j, b_k) = \frac{|\Gamma(b_j) \cap \Gamma(b_k)|}{|\Gamma(b_j) \cup \Gamma(b_k)|}$. Based on these remarks, the *personal interest value* of badge b_j for user u_i can be represented as

$$v^{pi}(u_i, b_j|\mathcal{H}) = \frac{\sum_{b_k \in \mathcal{H}} \frac{|\Gamma(b_j) \cap \Gamma(b_k)|}{|\Gamma(b_j) \cup \Gamma(b_k)|}}{|\mathcal{H}|}.$$

The effectiveness of the *personal interest value* of badge is evaluated with a similar experiment setting, whose result is available in Figure 6. We can observe that ranking badges according to their *personal interest values* for each user can achieve an AUC score of 0.66, and Precision@100 score of 0.41.

C. Network Steering Value Function

Besides the effects of personal interests and peer pressure, there exists some global trend about the network steering users badge achievement activities in the whole network. In online social network, users achieve badges in a sequential time order. For example, badges achieved by user u_i can be organized into a sequential transaction $\langle b_1^i, b_2^i, \cdots, b_l^i \rangle$ according to the achieving timestamps, where u_i got badge b_p^i before b_q^i if $p < q$. For all users in $\mathcal{U}$, we can represent the badge achievement sequential transactions as $\{u_1 : \langle b_1^1, b_2^1, ..., b_l^1 \rangle, u_2 : \langle b_1^2, b_2^2, ..., b_o^2 \rangle, \cdots, u_n : \langle b_1^n, b_2^n, ..., b_q^n \rangle\}$

The network influence can be captured by extracting the frequent badge achievement sequential patterns from the transactions. Consider, for example, we extract two frequent sequence patterns: pattern 1: $\langle b_l, b_o, \cdots, b_p \rangle$ and pattern 2: $\langle b_l, b_o, \cdots, b_p, b_q \rangle$ with supports $support$(pattern 1) and $support$(pattern 2) respectively from the network. Rule r can be generated based on pattern 1 and pattern 2 representing that for users who have obtained badges in $\langle b_l, b_o, \cdots, b_p \rangle$ has a chance of $conf$ to get badge b_q:

$$r : \langle b_l, b_o, \cdots, b_p \rangle \to \langle b_q \rangle, conf = \frac{support(\text{pattern 2})}{support(\text{pattern 1})},$$

where $\langle b_l, b_o, \cdots, b_p \rangle$ is called the antecedent of rule r (i.e., $ant.(r)$) and $\langle b_q \rangle$ is named as the consequent of r (i.e., $con.(r)$). Score $conf(r) = \frac{support(\text{pattern 2})}{support(\text{pattern 1})}$ is called the confidence of rule r. Various rules together with their confidence scores can be generated based on the frequent sequence pattern mining results, and the badge achievement activities fitting the rules can be modeled with the *network steering value* of the badges as follow.

Definition 8 (Network Steering Value Function): For a given user u_i, who has achieved a sequence of badges $\mathcal{H} = \langle b_{i_1}, b_{i_2}, \cdots, b_{i_{|\mathcal{H}|}} \rangle$ already, the *network steering value function* of badge b_j for u_i is defined as the maximal confidence score of rules that can fit historical badges in $\mathcal{H}$ and b_j, i.e.,

$$v^{ns}(u_i, b_j|\mathcal{H}) = \max\{conf(r)|r \in \mathcal{R}, ant.(r) \subset \mathcal{H}, con.(r) = b_j\}.$$

We evaluate the effectiveness of the introduced *network steering value* of badges based on the same experiment setting introduced in Section III-A2. As shown in Figure 6, *network steering value* based badge predictor along can achieve an AUC score of 0.68 and Precision@100 score of 0.45 in inferring potential badge achievement activities.

D. Comprehensive Badge Value Function

To capture the information from all the three aspects in calculating badge values, we define the *comprehensive value* value function as a combination of *personal interest value*, *peer pressure value* and *network steering value*.

Definition 9 (Comprehensive Value): Let the *personal interest value*, *peer pressure value* and *network steering value* of badge b_j to user u_i be $v^{pi}(u_i, b_j)$, $v^{pp}(u_i, b_j)$ and $v^{ns}(u_i, b_j)$ respectively. The *comprehensive value* of b_j to user u_i is defined as a combination of these 3 value functions:

$$v^c(u_i, b_j) = \alpha \cdot v^{pi}(u_i, b_j) + \beta \cdot v^{pp}(u_i, b_j) + (1 - \alpha - \beta) v^{ns}(u_i, b_j),$$

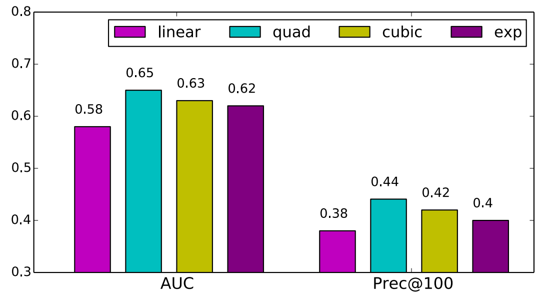


Fig. 5. Comparison of peer pressure value functions

where parameters α, β are assigned with value $\frac{1}{3}$ for simplicity in this paper.

Furthermore, to demonstrate the effectiveness of the above *comprehensive value* definition, we also compare it against the isolated value functions and the results is available in Figure 6. Here, for the *peer pressure value function*, the *quadratic* function is used as it can achieve the best performance in Figure 5. From the result we observe that *network steering value function* performs better than *personal interest* and *peer pressure* value functions, which can achieve AUC scores about 0.68, 0.66, and 0.65 respectively. Meanwhile, the *comprehensive value function* that merge the isolated value functions together can improve the performance greatly and can obtain AUC score is 0.77, which is 13.2%, 16.7%, and 18.5% higher than the AUC scores achieved by *personal interest*, *peer pressure* and *network steering* value functions respectively. Similar results can be observed when the evaluation metric is Precision@100, and the *comprehensive value function* can obtain Precision@100 score of 0.60.

IV. User Utility Function

Value of badges is the reward that users can receive from the system. Meanwhile, to get the reward, they also need to afford certain costs introduced when finishing the required tasks. Generally, if the reward is greater than the cost, the badge will deserve the efforts, and the payoff is called the *utility* formally. The formal definitions of the *reward*, *cost* and *utility* functions of badges for users are available in this section.

A. User Utility Function Definition

The *reward* user u_i can obtain by achieving badge b_j is defined as

$$reward(u_i, b_j) = I(c_{i,j} \geq \theta_j) v^c(u_i, b_j).$$

If u_i can obtain b_j, then the *reward* u_i can achieve will be the *comprehensive value* of badge b_j for u_i; otherwise, the reward will be 0. Meanwhile, to achieve a certain badge, e.g., b_j, the *cost* that u_i needs to pay is defined as the *cumulative effort* that u_i invests on b_j:

$$cost(u_i, b_j) = \hat{e}_{i,j}.$$

The minimum efforts $\tilde{e}_{i,j}$ required for user u_i to get badge b_j is determined by u_i's ability in achieving b_j as well as the badge threshold of b_j, which can be represented as

$$\tilde{e}_{i,j} = \arg\min_{\hat{e}} (a_{i,j}\hat{e}_{i,j} \geq \theta_j) = \frac{\theta_j}{a_{i,j}}$$

Definition 10 (Utility Function): The *utility function* of u_i in achieving b_j is defined as

$$\begin{aligned} utility(u_i, b_j) &= reward(u_i, b_j) - cost(u_i, b_j) \\ &= I(c_{i,j} \geq \theta_j) v^c(u_i, b_j) - \hat{e}_{i,j}. \end{aligned}$$

If u_i can get b_j, i.e., $a_{i,j}\hat{e}_{i,j} \geq \theta_j$, then $utility(u_i, b_j) = v^c(u_i, b_j) - \hat{e}_{i,j}$; otherwise, $utility(u_i, b_j) = -\hat{e}_{i,j}$.

The *overall utility function* of users u_i over all badges in $\mathcal{B}$ is represented as

$$utility(u_i) = \sum_{b_j \in \mathcal{B}} utility(u_i, b_j).$$

Utility function considers both the values and costs introduced by the badges in online social networks, which provides a more comprehensive modeling of individuals' badge achievement activities.

V. Utility Maximization based Badge Achievement

A. Game Among Users

In social networks, every user wants to maximize his/her utility in achieving badges, while the value of different badges for certain user may also depend on other users social activities. As a result, the badge achieving activities in online social networks can form a game among users. In traditional game theory, all the agents (e.g., users in social networks) are all assumed to be *self-interested*. Here, "self-interested" doesn't necessarily mean that users tend to harm other users to maximize their payoff, as it can also include good things happening to other users as well.

Meanwhile, what users can do in the *game* is determined by their *game strategies*. A user's *game strategy* refers to the options that he chooses in a setting where the outcome depends not only on his own actions but also on the actions of other users. A user's *strategy* can determine the actions the user will take at any stages in the game. In badge systems, users *strategy* can cover various aspects of their social activities but, in this paper, we refer to the *strategy* of users as the way how they distribute their *cumulative efforts* for simplicity, i.e., user u_i's strategy $s_i = \hat{\mathbf{e}}_i$.

Given the user set $\mathcal{U}$, we can represent the strategies of all users in $\mathcal{U}$ except u_i as $\mathbf{s}_{-i} = (s_1, s_2, \cdots, s_{i-1}, s_{i+1}, \cdots, s_n)$. Thus we can write the strategies of all users in $\mathcal{U}$ as $s = (s_i, \mathbf{s}_{-i})$, where $s_k = \hat{\mathbf{e}}_k, k \in \{1, 2, \cdots, n\}$. Meanwhile, depending on users' unique game *strategies*, different kinds of social activities will be exerted in achieving badges, which can lead to different *utilities*.

Definition 11 (Strategy Utility Function): Given user u_i's and other users' strategies: s_i and $\mathbf{s}_{-i}$, the *utility* that u_i can get based on s_i and $\mathbf{s}_{-i}$ can be represented as:

$$u(s_i, \mathbf{s}_{-i}) = utility(u_i | s_i, \mathbf{s}_{-i}) = \sum_{j=1}^{m} utility(u_i, b_j | s_i, \mathbf{s}_{-i}).$$

Different game strategies will lead to different game utilities. For users in online social networks, they all want to determine the optimal strategies to distribute their efforts to achieve the maximum utilities, and the optimal strategy is also formally called the *dominant strategy* in game theory. Let s_i and s'_i be two *strategies* of user u_i and $\mathbf{s}_{-i}$ be the strategies of all other users in $\mathcal{U}$ except u_i. The relationships between strategies s_i and s'_i can be categorized as follows:

- *Strict Domination*: for u_i, s_i *strictly dominates* s'_i iff $u(s_i, \mathbf{s}_{-i}) > u(s'_i, \mathbf{s}_{-i})$ for $\forall \mathbf{s}_{-i} \in S_{-i}$, where S_{-i} represents the set of all potential strategies of the other users;
- *Weak Domination*: for u_i, s_i *weakly dominates* s'_i iff $u(s_i, \mathbf{s}_{-i}) \geq u(s'_i, \mathbf{s}_{-i})$ $\forall \mathbf{s}_{-i} \in S_{-i}$ and $\exists \mathbf{s}_{-i} \in S_{-i}$, such that $u(s_i, \mathbf{s}_{-i}) > u(s'_i, \mathbf{s}_{-i})$;
- *Very Weak Domination*: for u_i, s_i *very weakly dominates* s'_i iff $u(s_i, \mathbf{s}_{-i}) \geq u(s'_i, \mathbf{s}_{-i})$ for $\forall \mathbf{s}_{-i} \in S_{-i}$.

Based on the above remarks, strategy s_i is a (strictly, weakly, very weakly) *dominant strategy* iff s_i can (strictly, weakly, very weakly) *dominate* s'_i for $s'_i \in S_i, s'_i \neq s_i$, regardless of other users' strategies (i.e., $\mathbf{s}_{-i}$). The optimal distribution of u_i's *cumulative efforts* is identical to the *dominant strategy* of u_i, which can be obtained by solving the following *maximization objective function*:

$$\hat{s}_i = \arg\max_{s_i} u(s_i, \mathbf{s}_{-i}),$$

where $\hat{s}_i$ is the *dominant strategy* of u_i and other users strategies $\mathbf{s}_{-i} \in S_{-i}$ can take any potential value.

The above objective function is very hard to solve mathematically, as we may need to enumerate all potential strategies of all the users (including both u_i and other users) in the network to obtain the global optimal strategy of u_i. Based on the assumption that all users are "self-interested", in this paper, we propose to calculate the equilibrium state of all users strategy selection process instead as follows:

We let the users to decide their optimal strategies in a random order iteratively until convergence. At first, in the 1_{st} round, we let users to decide their optimal strategies in a random order. For example, if we let u_i be the first one to choose his "optimal strategy" when other users are not involved in the system (i.e., $\mathbf{s}_{-i} = \mathbf{0}$), we can represent strategy selected by u_i's as:

$$\tilde{s}_i = \arg\max_{s_i} u(s_i, \mathbf{0}).$$

Based on u_i's "optimal strategy", other users in $\mathcal{U} - \{u_i\}$ (e.g., u_j) will take turns to decide their own "optimal" strategies by utilizing the selected strategies of other users. For example, let u_j be the 2_{nd} user to decide his/her strategy right after u_i. The "optimal strategy" of u_j can be represented as

$$\tilde{s}_j = \arg\max_{s_j} u(s_j, \{\tilde{s}_i\} \cup \mathbf{0}).$$

And let u_k be the last user to select the "optimal strategy" in the 1_{st} round. Based on the known strategies selected by all the other users, the "optimal strategy" of u_k can be represented as

$$\tilde{s}_k = \arg\max_{s_k} u(s_k, \{\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_{k-1}, \tilde{s}_{k+1}, \cdots, \tilde{s}_{|\mathcal{U}|}\}).$$

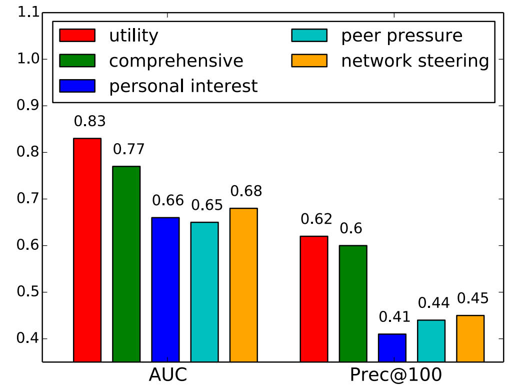


Fig. 6. Comparison of utility maximization based badge achievement strategy with comprehensive value function and other isolated value functions

After finishing the 1_{st} round, we will start the 2_{nd} round and all users will decide their strategies in a random order. Such a process will continue until all users' "optimal strategies" selected in round k is identical to those in round $k-1$ (i.e., the stationary state), which will be outputted as the final optimal strategies of all users.

B. User Game Strategy Evaluation

To demonstrate the effectiveness of game theory in modeling users' badge achieving activities, we conduct the experiments to show the performance of *utility based user game* in inferring users badge achieving activities. Experiment settings here is identical to those introduced before but, to calculate the utilities of different badges for users, we need to know users' total cumulative efforts, ability distributions, and badge thresholds in advance.

Inference of Cumulative Effort: Active users in online social networks are assumed to have more *cumulative efforts*. In our dataset, the activeness measure can be defined as the number of badges users achieved. And the cumulative effort of user, e.g., u_i, can be obtained by normalizing the badge numbers to the range of $[0, 1]$ with equation $\frac{\#(u_i) - \#min}{\#max - \#min}$, where $\#(u_i)$ is the number of badges achieved by u_i and $\#max$ and $\#min$ are the maximal and minimal number of badges achieved by users in $\mathcal{U}$ respectively.

Inference of User Ability Vector: In the training set, user u_i's inferred ability vector is defined to be $\mathbf{a}_i^{infer} = (a_{i,1}, a_{i,2}, \cdots, a_{i,m})$ of length $m = |\mathcal{B}|$, where $a_{i,j}$ is the number of times that u_i obtained badge of category b_j in the training set. Each user is assumed to have the same amount of ability but can be distributed differently. Vector $\mathbf{a}_i^{infer}$ is normalized by the total number of achieved badges to ensure $\left|\mathbf{a}_i^{infer}\right|_1 = 1$. Considering that users can have their hidden abilities, a random ability vector $\mathbf{a}_i^{random}$ of length m is generated whose cells contain random numbers in $[0, 1]$ and $\alpha \cdot \mathbf{a}_i^{infer} + (1.0 - \alpha) \cdot \mathbf{a}_i^{random}$ is used as the final ability vector of user u_i. In this paper, we set parameter $\alpha = 0.85$.

Inference of Badge Threshold: Badges which are hard to achieve will be obtained later. For each badge $b_j \in \mathcal{B}$, we get all the users who have achieved b_j from the training set: $\{u_1^j, u_2^j, \cdots, u_k^j\}$. For user $u_i^j \in \{u_1^j, u_2^j, \cdots, u_k^j\}$, we organize all the badges obtained by u_i^j from the training set in a sequence according to their achieving timestamps, the index of b_j in u_i^j's achieved badge list is extracted to calculate b_j's threshold. For example, if u_i have achieved p badges in all and the index of b_j in the list is q, then the threshold of b_j for u_i is estimated as $\theta_{j,i} = \frac{p}{q}$. The threshold of badge b_j is defined as the average of thresholds calculated for all these users: $\theta_j = \eta_j \frac{\sum_{o=1}^{k} \theta_{j,o}}{k}$ where η_j is a scaling parameter. Value η_j is selected as large as possible but, at the same time, η_j needs to ensure that for all users who have obtained badge b_j in the training set (i.e., $\forall u_i^j \in \{u_1^j, u_2^j, \cdots, u_k^j\}$). When u_i^j devotes all his cumulative effort to get b_j, u_i^j's contribution can obtain b_j in our model and, in other words, his contribution can exceed θ_j.

Based on the above inferred *cumulative efforts*, *ability* of users as well as *badge thresholds*, we can perform the game among the users in achieving badges. The results achieved by the *utility maximization based badge achievement strategy* are shown in Figure 6. From the results, we can observe that the introduced *user utility maximization based game strategy* can perform very well in modeling users badge achieving activities. The AUC score achieved by the *utility maximization based badge achievement strategy* is 0.83, which is 7.8% larger than the AUC score achieved by *comprehensive value function* (i.e., 0.77). Similarly, the Precision@100 achieved by the *utility maximization based badge achievement strategy* is 0.62, which is larger than the other comparison value functions. As a result, *utility maximization based badge achievement strategy* can provide a more comprehensive modeling about users' badge achievement activities.

VI. Related Work

Reward systems, e.g., badge system, have been widely employed in online social networks, like Foursquare [3], [1], [4]. Antin et. al. study the badges in online social networks from a social psychological perspective and give some basic introduction of badges in Foursquare [3]. Large amount of badges are placed in Foursquare and a complete list of Foursquare badges is available [1]. To obtain badges in Foursquare, users need to reveal their locations by checking in at certain locations. Carbunar et. al. study the problem between privacy preservation and badge achievement in Foursquare [4].

Users are assumed to be "selfish" and want to maximize their payoff, which will form a game among users in badge achievement. There has been a growing literature on analyzing the game among users in online social networks. Ghosh et. al. [9], [6], [8] provide a game-theoretic model within which to study the problem of incentivizing high quality user generated content, in which contributors are strategic and motivated by exposure. Jain et. al. [12] present a simple game-theoretic model of the ESP game and characterize the equilibrium behavior in their model. Their equilibrium analysis supports the fact that users appear to be coordinating on low effort words.

To achieve the maximal contribution to the sites, many works have been done on designing the badge system for online social networks. Jain et. al. [11] study the problem of incentive design for online question and answer sites. Anderson et. al. [2] study how badges can influence and steer users behavior on social networks, which can lead both to increased participation and to changes in the mix of activities a user pursues in the network. Ghosh et. al. [7] study the problem of implementing a mechanism which can lead to optimal outcomes in social computing based on a game-theoretic approach. Immorlica et. al. [10] study the badge system design whose goal is to maximize contributions. Easley et. al. [5] take a game-theoretic approach to badge design, analyzing the incentives created by badges and potential contributors as well as their contribution to the sites.

VII. Conclusion

In this paper, we study the badge system analysis problem. We introduce the three different categories of badges value functions for users in online social networks. To depict users' payoff by achieving badges in online social networks, we formally define the utility function for users. We solve the "badge system analysis" problem as a game among users in social network. Experiments conducted on real-world badge system dataset demonstrate that our model can capture users' motivations in achieving badges online very well.

VIII. Acknowledgement

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