DistSD: Distance-based Social Discovery with Personalized Posterior Screening

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Abstract—Privacy preservation in location-based proximity services has recently received considerable attention in geo-social networks. Nearby friends notification and social discoveries are two important types of location-based proximity services. A large number of privacy protection methods have been proposed on nearby friends notification, but few on social discoveries. Most of existing protection methods on nearby friends notification cannot be applied in social discoveries, since a secret key needs to be shared between dynamic friends. In this paper, we address the research challenges that the location privacy protection in distance-based social discoveries. We propose a novel framework DistSD for the distance-based social discovery with personalized posterior screening. We also show that the problem that finding an optimal safe group of nearby seeking users is NP-hard. Two heuristic privacy enhanced social discovery algorithms are proposed, which protect users’ locations from a service result perspective. Experiments are conducted based on the real-life data and experimental results validate the effectiveness and efficiency of the proposed algorithms.

I. INTRODUCTION

With the advance of location-acquisition and mobile communication technologies, location based proximity services have become increasingly prevalent. Location-based proximity service is a distance preserving service, notifying users about other nearby users and showing their distances. According to the strength of social relation in the social networks, location based proximity services are classified into contact-based nearby friends notifications (e.g., Swarm, Facebook Place, PCube, etc.) and distance-based social discoveries (e.g., Wechat, Tinder, Momo, etc.). Many of these services have already been adopted and used by millions of users, and the number keeps growing steadily.

Contact-based nearby friends notifications (called nearby friends for short) are based on the prerequisite that two users are friends mutually. However, distance-based social discoveries (called social discoveries for short) establish the connection of two users based on the physical proximity, even though the two users may not know each other before. For example, assume that a new user just creates an account on Tinder, a popular mobile dating service. The profile of the new user, such as sexual orientation, marital status, salary etc., is visible to any Tinder user who is in the proximity instantly. Therefore, privacy protection is critically needed for social discoveries.

In this paper, we focus on the location privacy protection in social discoveries. Unfortunately, the existing social discoveries are risking the user’s location privacy. It is reported that some government had used a mobile dating app to locate and imprison gay users [11]. It has also been proved theoretically and experimentally that social discoveries can render users vulnerability to trilateration attacks and distance-free locating attacks [7], [4], [3], [11]. More sadly, employing the current workflow, it is impossible to provide a safe social discovery service for each user. From the perspective of data management, the existing social discovery service is a typical scenario for k nearest neighbors queries in a constrained space (i.e., the proximity region specified by a user). It has been proved in the recent work [16] that the existing methods for secure nearest neighbor (SNN) queries [6], [14] are not secure, and it is impossible to construct SNN in standard security models [2].

We use an example in Fig. 1 to illustrate the case of location disclosure in social discoveries. Assume that \( u_1, u_2, \) and \( u_3 \) are social discovery app users (e.g., mobile dating). \( u_1 \) is interested in \( u \) but \( u \) refuses to meet \( u_1 \) in person, \( u_1 \) asks his friends \( u_2 \) and \( u_3 \) for help, and \( u \) is shown to be nearby to these three malicious users (i.e., \( u_1, u_2, \) and \( u_3 \)) concurrently. By utilizing the precise locations and proximity distances of \( u_1, u_2, \) and \( u_3, \) they can infer that \( u \) must locate within the gray area (i.e., the shared proximity regions by \( u_1, u_2, \) and \( u_3 \)) in Fig. 1. If \( u_1, u_2, \) and \( u_3 \) continue to change their locations, the gray area can shrink to an exact location. A software agent has been developed in [4] to conduct the above attack automatically. Users in a famous social network was pinpointed within 5 meters of their exact locations [11]. An exact location exposure can cause serious problems, such as unwanted advertisement, location-based spams/scams, and even stalking. To differentiate the roles of user \( u \) from \( u_1, u_2, \) and \( u_3, \) \( u \) is called a discovered user, while \( u_1, u_2, \) and \( u_3 \) are called seeking users. Generally, a user can be a discovered user, a seeking user, or both.

In order to protect user’s location in social discoveries, we propose a novel framework DistSD (Distance-based Social Discovery with Personalized Posterior Screening). The intrinsic of DistSD is that a discovered user can show up in front of seeking users selectively, instead of being discovered passively. We illustrate our basic idea with the same example in Fig. 1.
We propose a novel framework DistSD, in which a user can determine whom she may share her proximity with, rather than automatically give the location and thus leak private information.

We propose two algorithms FBuck and LCF\textsuperscript{2} in mobile devices to achieve a balance between QoS and location privacy protection.

We propose a posterior screening mechanism in distance-based social discoveries, which aims to protect the location privacy on the basis of service results.

A series of experiments is conducted on a real dataset to evaluate the performance of our proposed algorithms.

The rest of the paper is organized as follows. We review the related work in Section II. The new framework DistSD is given in Section III. Section IV formally defines the problem. Section V sketches out the basic idea of the proposed algorithms. Two privacy-enhanced social discovery algorithms FBuck and LCF\textsuperscript{2} are proposed in Section VI and Section VII respectively. Section VIII presents the performance evaluation results. Finally, the paper is concluded in Section IX.

II. RELATED WORK

A great of research efforts [8], [10], [1], [12] have been devoted in investigating location privacy protection, that preserves the privacy of mobile users and ensures the high quality of LBSs. We categorize the existing location privacy protection mechanisms into a priori protection and a posterior screening. In a priori protection mechanism, an actual location is replaced with an obfuscated location before the query is issued. The mainstream idea for location obfuscation [8], [10], [1] includes generalization (e.g., spatial cloaking), cryptography (e.g., PRI), generating dummies, and adding noise (e.g., geoindistinguishability). A priori privacy protection has been a very active research area in points of interests retrieval in LBSs in the last decade. In contrast, in a posterior screening mechanism, the user’s location is protected from a query result perspective after the query is answered. The obfuscated location for each user is an uncertain region in our method. However, the obfuscated location is computed from the service results (i.e., nearby records) instead of being sent by the user themselves. Thus, our work falls into the category of a posterior screening.

Privacy protection on nearby friends has been extensively studied in recent years [9], [13]. The basic idea is to allow two friends to determine if they are in proximity based on both spatial cloaking and encryption, which falls into the category of a priori protection. One drawback of the existing methods is that a proximate distance and a distance ranking list between friends are revealed, which can result in location disclosure [7]. Another one is that each user needs to share a symmetric secret key with each of her friends. However, all nearby users are considered as friends in social discoveries. The set of friends changes dynamically, and the number of potential friends for each user is large. Sharing a secret with each friend dynamically seems to be impractical [9]. Thus,
existing privacy protection methods on nearby friends cannot be applied in social discoveries.

III. FRAMEWORK OF DistSD

We employ a client-server architecture, including users and SPs. None of the entities in the service space is trusted. Thus, like most existing work [16], [6], we assume SPs and users are semi-honest, who are honest but potentially curious. Since the SP is un-trusted, the form of a user’s obfuscated location owned by the SP has two options, i.e., an encrypted location or an uncertain region. Users may prefer encrypted locations, which indicate a strong privacy protection. However, to the best of our knowledge, secure reverse range queries, that the problem of finding encrypted regions where a user locates, is still an open issue. Thus, a safe uncertain region is used as a user’s obfuscated location in this paper. Note that, the obfuscated location of a user is computed by the SP from nearby records instead of being sent by the user herself.

SPs store users’ registration information and seeking users’ PRs. Each PR is extended from the user’s obfuscated location. The SP organizes PRs as a grid based index (details in Section VI-A and Section VII-A). Before using the social discovery service, each user personalizes the minimum size of obfuscated locations at the mobile device. Only the user herself knows the size. The minimum uncertain size indicates the maximum acceptable area, which the user allows other un-trusted entities to know, e.g., SPs or curious friends. Users in DistSD (i.e., mobile devices) download a fraction of PRs of seeking users periodically. When a user uses the social discovery service, the user aims to meet local new friends, that implies a seeking user will stay at a location for a while. Thus, we assume seeking users don’t update locations frequently.

When a user opens a social discovery app, a list of selective nearby seeking users, whose PR covers the user’s location, is recommended to the user by running any of the proposed methods in mobile devices. The user’s location privacy is posterior screened from the recommended seeking users. The proximity information in the form of nearby records is sent to the SP automatically. Based on the nearby records, the SP notifies the corresponding seeking users that the discovered user is nearby. Then, an obfuscated location of each discovered user is inferred by the SP as the attack scenario described in Fig. 1. The safety of the obfuscated location has been verified in mobile devices (i.e., in our proposed methods) beforehand. The SP extends the obfuscated location to a PR as the user’s registration information. We use a rectangle for representing a PR to simplify computation. The grid based index is updated when a new seeking user with her correspond PR is inserted. Each user has a valid time interval $T$, which is a system parameter. If a user has no actions within $T$, e.g., being notified or chatting, the user’s log-in information will expire and be deleted from the SP.

Note that a PR is created from the user’s nearby records indirectly. At the initial phase, the system will encounter a cold startup problem. Since none of the users has nearby records initially, the SP can get neither obfuscated locations nor PRs. We resolve this problem in two ways. One method is to employ venues check-in records in check-in services, where a check-in record corresponds to a proximity record. Many nearby friends services and venues check-in services are combined in one app, such as Foursquare, Google+, and etc. It is possible to borrow venues check-in records to start the system. The other method is to employ a prior privacy protection method to obtain an obfuscated location, which is prevented from background knowledge attacks [12].

In summary, the work flow of DistSD is as follows. At the client side: (1) The user specifies a minimum size of an obfuscated location as the privacy requirement in the mobile device. (2) A fraction of seeking users’ PRs are downloaded periodically. (3) A list of posterior-privacy-screened seeking users is recommended. The user can select to notify the seeking user manually or automatically. (4) The nearby records are sent to the SP. At the SP side: (1) The SP notifies the seeking users in the nearby records that the discovered user is nearby. (2) The SP infers an obfuscated location for each discovered user from the nearby records. (3) When a discovered user changes to be a seeking user, a PR is computed based on the obfuscated location, and the grid based index is updated. (4) When a user has no actions for $T$, the user is deleted from DistSD.

Safety analysis: In our work, the protection target is the user’s locations. Most of social discovery services require users to use real name for safety considerations. Thus, we don’t consider identity anonymization. From the SP side: The locations of a user, that SPs have, include both an obfuscated location and a PR. Since the latter one is extended from the former one, the obfuscated location is the most accurate location that the SP knows. Recall that an obfuscated location is computed from the nearby records of the user. The safety of the obfuscated location has been ensured with posterior screening mechanism in our proposed methods before the nearby records are sent to the SP. Thus, the location privacy is protected from the semi-honest SPs.

From the users side: For a discovered user, the worst case is that all the notified seeking users collude with each other. In such a case, the information that the corrupted users can gather is just the nearby records known by the SP. The exact location of the discovered user can neither be broken down by the SP as the above analysis, nor by the corrupted seeking users. Meanwhile, the seeking user’s own locations cannot help them to reduce the uncertainty of the discovered user’s location. Thus, the accurate location of a discovered user is protected from seeking users. On the other hand, if a discovered user is malicious, the discovered user can only get the PR of a seeking user, which satisfies the seeking user’s privacy requirement. In another word, no one knows a user’s precise location in DistSD except the user herself.

IV. PRELIMINARY

A. Attack model

Definition 1. (Footprints) A set of user $u$’s nearby records is formalized as $\{\langle u, u_1 \rangle, \langle u, u_2 \rangle, \ldots, \langle u, u_n \rangle\}$. $\langle u, u_i \rangle$ indicates
B. Problem Definition

Recall the attack scenario in Section I, we observe that over-share discovery attacks may be satisfied. The analysis of obfuscated location of a user will become larger when a user notifies a subset of Upt. In the above example, if u notifies {u1, u3}, or {u1}, etc., minu may be satisfied. The objective of our problem can be formally described as follows.

**Goal:** We aim to find a users set sb that a discovered user u, such that (1) sbu ⊆ Upt; (2) size(ALSbu) ≥ minu; (3) ∀s′bu ∈ Upt (s′bu ≠ sbu) and size(ALSbu′) < minu, |sbu′| ≥ |sbu|, where |sbu| is the number of users in sbu. The third condition indicates sbu is one of the largest subsets of Upt that satisfies the first two conditions.

**Theorem 2.** Finding a maximal subset sbu in Upt with the constraint size(ALSbu) ≥ minu is NP-complete.

Our problem can be exactly mapped to a classical frequent item set selection problem by treating Upt as the complete item set and sbu as the selected subset (whose occurrence frequency is size(ALSbu)). minu corresponds to the minimum frequency requirement. The problem of counting the number of the maximal minu-frequent itemset in Upt has been proven to be #P-complete in [15], i.e., the corresponding frequent itemset mining problem is NP-complete. Therefore, our problem is also NP-complete. For convenience, we list the interpretations of primary symbols throughout the paper in Table I.
V. PROPOSED ALGORITHMS SKETCH

In order to achieve the above goal, our basic idea is to divide the task into two parts. One part is to organize PRs on the SP into an index, which is downloaded by mobile devices periodically. With the help of a PR index, a mobile user can find the covered PRs $FS_u$ efficiently. The complete contents of a user in SPs include numerics, texts, images, graphs (e.g., friends circles), and even videos, whose size could be significant. It is impractical to download the complete contents due to the constrained network bandwidth, device battery, etc. Thus, we propose two grid-based indices (i.e., HGrid and DGrid in Section VI-A and Section VII-A respectively) to organize the PRs in SPs.

The other part is to find a maximum safe footprints in mobile devices, which is the core of our proposed methods. The basic process of finding a maximum safe footprints is as follows. With the index downloaded from SPs and the user's own exact location, the users $FS_u$ whose PR covers the mobile client's location are found. Then, from this subset of $FS_u$, the corresponding intersection area of their PRs are computed. Next, the intersection area is verified by the user's uncertainty requirement $min_u$. If $min_u$ is not satisfied, another subset of $FS_u$ is checked. Finally, a maximum subset of $FS_u$ whose PRs intersection area is not less than $min_u$ is returned as the safe footprints.

Baseline: The naïve method for finding the maximum safe subset of $FS_u$ is the brute-force calculation. Specifically, we enumerate each user combination in $FS_u$ with decreasing size of users set, and compute the intersection area of their PRs. As we know, enumeration each combination of a users set is costly. As the number of seeking users in $FS_u$ increases (e.g., several thousands in our experiments), the efficiency of baseline will deteriorate. In order to improve the efficiency, we propose two grid-based algorithms: a global algorithm FBuck (in Section VI-B) and a local algorithm LCF$^2$ (in Section VII-B).

VI. FBUCK: GLOBAL FOOTPRINTS FINDING ALGORITHM

A. HGrid

A HGrid on the SP contains a grid and a hash table. The grid just stores the seeking users identities, which helps mobile clients to find the seeking users whose PR covers the client's location. Each cell in the grid maintains two linked lists: (1) $pt$: a list that tracks the users whose PR covers the cell partly. (2) $ft$: a list that stores the users whose PR covers the whole cell. Meanwhile, a hash table saves the location of a user's PR. Each item in the hash table is $(id, x_1, y_1, x_2, y_2)$, where $id$ is the seeking user identity, $(x_1, y_1)$ and $(x_2, y_2)$ are bottom-left and top-right coordinates of the user's PR respectively.

B. FBuck

Incorporating with the HGrid and the user location $(x, y)$, the users set $FS_u$ whose PR covers $(x, y)$ are found. Different from Baseline, we employ top items in four buckets to compute the intersection area for each user combination from $FS_u$.

We first show the data structure of the bucket, and then give the main idea of the algorithm. For simplicity, we name four borders of a rectangle as left, right, bottom and top. Each bucket represents one kind of PR bounds, that is, a left bucket $l_{buk}$, a right bucket $r_{buk}$, a bottom bucket $b_{buk}$, and a top bucket $t_{buk}$. The item in each bucket includes a user $id$ and a boundary coordinate $pos$ of the PR on the x-axis (for $l_{buk}$ and $r_{buk}$) or the y-axis (for $b_{buk}$ and $t_{buk}$). Using $pos$ as the key, $l_{buk}$ and $b_{buk}$ are sorted non-increasingly, and $r_{buk}$ and $t_{buk}$ are sorted non-decreasingly. Fig. 3(a) shows an example of four buckets for three users’ PRs.

We observe that the top items in the four buckets constitute the smallest intersection rectangle (i.e., a pure rectangle) at present. Based on this observation, the main idea is as follows. The top items in the four buckets constitute a rectangle $ur$. If $ur.area < min_u$, a user $du$ is selected among the four top items, and is deleted from each bucket. The selection strategy for user removing is in the next subsection. The new top items shape into a new intersection rectangle. If the new rectangle area is still less than $min_u$, the above steps are repeated until every bucket becomes empty or the area is not less than $min_u$.

Algorithm 1 FBuck

1: find the covering users $FS_u$ in the cell $c$;
2: init $l_{buk}, r_{buk}, b_{buk}, t_{buk}$ by the PR of each $u_i$ in $FS_u$;
3: sort items in $l_{buk}$ and $r_{buk}$ non-increasingly;
4: sort items in $b_{buk}$ and $t_{buk}$ non-decreasingly;
5: while $l_{buk}$ is not empty do
6: get the top item from each bucket;
7: a rectangle $ur$ is formed from the top items;
8: if $ur.area < min_u$, then
9: $du$=the user selected to be removed;
10: delete $du$ from the four buckets and $FS_u$;
11: re-sort $l_{buk}$, $r_{buk}, b_{buk}, t_{buk}$;
12: else
13: return remaining users in $FS_u$ as the footprints;

Fig. 3 shows a running example of FBuck. The user $u$ (i.e., the small triangle in Fig. 3) is covered by the PRs of $\{u_1, u_2, u_3\}$. Initially, the top items in the four buckets constitute a rectangle ABCD. The users in the current top items are $\{u_1, u_2, u_3\}$. If the area of ABCD is less than $min_u$, $u_3$ is assumed to be deleted from each bucket. Then, each bucket is shown in Fig. 3(b) after $u_3$ is deleted. Now, the new top items constitute a new rectangle EFGC. If the area is not less than $min_u$, $\{u_1, u_2\}$ is returned as the safe footprints set.

1) Removing Users Selection: Our basic user removing idea is to delete the user whose PR borders limit or will limit the size of the obfuscated location, such that the uncertain requirement will be satisfied as early as possible. Before specifying the selective metrics, we define two distances (i.e., inner distances and outer distances) between a discovered user $u$ and a seeking user $u_i$.

Definition 5. (Inner Distance) Let $FS_u$ be a seeking users set, $pt$ be the intersection rectangle of PRs of users in $FS_u$,
and the boundary users of pt be $FS_{u}^{b}(\subseteq FS_{u})$. Assume that a discovered user $u$ locates in pt. For $\forall u_{i} \in FS_{u}$, the inner distance

$$indist(u, u_{i}) = \begin{cases} d_{u}, & \text{if } u_{i} \in FS_{u}^{b} \\ \infty, & \text{if } u_{i} \in FS_{u} - FS_{u}^{b} \end{cases}$$

(1)

where $d_{u}$ is the minimum perpendicular distance from $u$ to the pt boundary determined by $u_{i}$.

A small $indist(u, u_{i})$ implies one of $u_{i}$ PR’s boundaries limits the size of pt. If a user $u_{i}$ with the small inner distance is removed earlier, the intersection area of PRs tends to become larger.

Definition 6. (Outer Distance) $p_{u}^{bl}$, $p_{u}^{br}$, $p_{u}^{tl}$, $p_{u}^{tr}$ are the four corner points of $u_{i}$’s PR ($u_{i} \in FS_{u}$), that is the bottom-left corner; the bottom-right corner, the top-left corner and the top-right corner respectively. For $\forall u_{i} \in FS_{u}$, the outer distance

$$outdist(u, u_{i}) = \underset{p \in (p_{u}^{bl}, p_{u}^{br}, p_{u}^{tl}, p_{u}^{tr})}{\operatorname{MIN}} (dist_{pu})$$

where $dist_{pu}$ is the distance between the location of $u$ and the point $p$.

A small $outdist(u, u_{i})$ indicates $u_{i}$’s PR may limit the size of the obfuscated location later. In other words, a small $outdist(u, u_{i})$ implies a small available space to increase the intersection area. Thus, the users with small outer distances are expected to be removed earlier. We take $u$ and $u_{i}$ in Fig. 3(b) as an example. $indist(u, u_{i}) = d_{1}$, and $outdist(u, u_{i}) = d_{2}$.

The options for selecting a removing user (line 9 in Algorithm 1):

Option 1: A user with the minimum inner distance $indist$ is removed.

Option 2: A user with the minimum outer distance $outdist$ is removed.

Option 3: A user with the minimum $indist \times outdist$ is removed.

VII. LCF2: LOCAL CELL FOOTPRINTS FINDING ALGORITHM

In FBuck, the whole PRs are downloaded by mobile clients. The communication cost can be high. We observe that a city is usually divided into independent districts on the basis of different urban functions, geographical position, etc., which is far greater than location uncertain requirements. For example, Beijing contains 16 districts. In order to reduce the communication cost, we propose LCF2, which downloads a part of PRs in a district, and finds a safe footprints set from local PRs. The problem of how to divide a city is out of our scope.

A. DGrid: A Grid with Corner-Rect Departure Cells

We still use a grid to index the PRs. However, the PRs’ location information is stored in the cells of the grid.

1) Cells of a DGrid: Before elaborating the feature of the DGrid, we first define the cell size. We observe that a closed region is formed when a left $l$ meets a right $r$ on the x-dimension, meanwhile, a bottom $b$ meets a top $t$ on the y-dimension. This phenomena still holds when the four borders belong to different users. The cell size is defined as the minimum distance, at which an $l$ meets an $r$ or a $b$ meets a $t$ when x-coordinates and y-coordinates increase respectively. All PR’s x-coordinates (y-coordinates) of $l$ and $r$ ($b$ and $t$) constitute a set $PR_{l}$ and $PR_{r}$ ($PR_{b}$ and $PR_{t}$) respectively. $PR_{l}$, $PR_{r}$, $PR_{b}$ and $PR_{t}$ are sorted ascendingly.

Definition 7. (Cell Size) The cell size $\delta$ of a grid $G$ is $\delta = MIN(\delta_{x}, \delta_{y})$, where $\delta_{x} = MIN_{\forall x_{i} \in PR_{l}, x_{i} \in PR_{r}} (x_{r} - x_{l})$, $\delta_{y} = MIN_{\forall y_{i} \in PR_{b}, y_{i} \in PR_{t}} (y_{t} - y_{b})$. $G$ is called a DGrid.

For a cell c in a DGrid, if an $r$ and an $l$ cross c at the same time, the $r$ must locate at the left of $l$. Similarly, if a $t$ and a $b$ are in $c$ simultaneously, the $t$ must be below the $b$. In Fig. 4, the dash square represents a cell. Fig. 4(a) shows the cases when four kinds of PR’s borders cross c simultaneously, and Fig. 4(b) shows a case which never occurs in a DGrid.

A corner rectangle is formed when a PR intersects with a cell. For instance, a bottom-right corner rectangle ($br$-rect) is formed when a PR intersects with a cell partly by a PR bottom and a PR right. As shown in Fig. 4 (a), the corner rectangles (i.e., $br$-rects, $bl$-rects, $tr$-rects, and $tl$-rects) in a cell depart from each other. This property is formalized in Theorem 3.

Theorem 3. For any cell c in a DGrid G, if the right of a PR $pr_{u}$ and the left of another PR $pr_{u}'$ cross c simultaneously, $pr_{u}.x_{r} \leq pr_{u}'.x_{l}$; otherwise, the right of $pr_{u}$ and the left of $pr_{u}'$ must locate at the right and left boundaries of c respectively, where $pr_{u}.x_{l}$ and $pr_{u}.x_{r}$ are the x-coordinates of $pr_{u}$’s left and right respectively. Similarly, if the top of $pr_{u}$ and the bottom of $pr_{u}'$ cross c simultaneously, $pr_{u}.y_{t} \leq pr_{u}'.y_{b}$; otherwise, the top of $pr_{u}$ and the bottom of $pr_{u}'$ must locate at the top and bottom boundaries of c respectively, where $pr_{u}.y_{b}$ and $pr_{u}.y_{t}$ are the y-coordinates of $pr_{u}$’s bottom and top respectively.

Theorem 3 can be proved by contradiction. Due to space limitation, the proof is omitted.

Definition 8. ($m$-rects) An $m$-rect is the intersection of a user’s PR and a cell c. $m$ is the number of $m$-rect boundaries determined by a user’s PR. From Theorem 3, $m \in \{0, 1, 2\}$. The other $4 - m$ boundaries are cell borders.
If \( m = 0 \), the PR fully covers the cell \( c \); otherwise, the PR partly covers \( c \). Fig. 5 (a) shows a DGrid with three PRs. The \( m \)-rect in the cell \((6, 6)\) is a 0-rect, whose borders are all the cell’s boundaries. On the other hand, a rectangle, whose borders are all the boundaries of user’s PRs, is called a user rectangle. A cell contains several \( m \)-rects.

**Definition 9. (Information in a Cell)** Let \( G \) be a DGrid. The information in each cell \( c \in G \) is formalized as \((\text{pos}, \text{rects})\).

\[ \text{pos} = (c_x, c_y) \]

is the location index of the cell in \( G \). \text{rects} is a collection of \( m \)-rects: \((\text{ty}, \text{ids}, \text{dist})\).

- If \( m = 0 \), \( \text{ids} \) are user identities whose PR covers \( c \) fully. \( \text{ty} \) and \( \text{dist} \) are both null.
- If \( m = 1 \), \( ty \in \{l, r, b, t\} \) represents a left, right, bottom or top border of PR crosses \( c \); \( \text{ids} \) are the users whose PR’s \( ty \) boundary crosses \( c \); \( \text{dist} \) is the width wid of \( m \)-rect if \( ty \in \{l, r\} \) or the height hgt of \( m \)-rect if \( ty \in \{b, t\} \).
- If \( m = 2 \), \( ty \in \{bl, br, tl, tr\} \) represents a bottom-left, a bottom-right, a top-left, or a top-right corner rectangle in \( c \), as Fig. 4 (a) shows. \( \text{ids} = (id_l, id_u) \), where \( id_l \) and \( id_u \) are users whose PR’s left or right (bottom or top) crosses \( c \). \( \text{dist} = (\text{wid}, \text{hgt}) \), where \( \text{wid} \) and \( \text{hgt} \) is the width (height) of \( m \)-rect.

In Fig. 5 (a), the cell \((5, 7)\) has three \( m \)-rects, i.e., an \( l \)-rect, a \( t \)-rect, and a \( tl \)-rect.

2) **Inserting and Deleting Users:** When a user \( u \) is deleted from a DGrid \( G \), the cells which are covered by \( pr_u \) are found. Then, \( u \) is deleted from the corresponding \( m \)-rects in the cells. When a new user \( u \) is inserted into \( G \), the insertion methods for fully covered cells and partly covered cells are different. For fully covered cells, \( u \) is inserted into 0-rects. For partly covered cells, there is a case that an \( (a) \) \( l \) (\( b \)) meets an \( (a) \) \( r \) (\( l \)) in \( c \), which will violate Definition 7. In this case, the affected left (right, bottom, or top) of a user’s PR is shifted to the left (right, bottom, or top) boundary of the cell. In other words, an affected PR border is extended by no more than \( \delta \). Then, the user is inserted into the corresponding \( m \)-rects. The detailed insertion and deletion algorithms are omitted due to the space limit.

**B. LCF\(^2\)**

LCF\(^2\) employs the \( m \)-rects in local cells to find the safe footprints. The basic idea is to extend the \( m \)-rect in the cell, which the user is in, by the proper \( m \)-rects in neighbor cells, such that a safe user rectangle is formed. The users related to the safe user rectangle are the safe footprints.

The main procedure is as follows. LCF\(^2\) starts from a tight-est \( m \)-rect where the user is (details in Section VII-B1). The \( m \)-rect is extended to be a user rectangle \( ur \) by applying an extending rule (details in Section VII-B2). If \( ur \).area < \( \min ur \), \( ur \) is inserted into a stack \( st \). While \( st \) is not empty, the top item \( tm \) is got. According to a boundary selection rule (details in Section VII-B3), one border of \( tm \) is opened. According to the extending rules at opening points (details in Section VII-B2), a new user rectangle \( nur \) is found. If \( nur \).area > \( \min nur \), the users related to \( nur \) are returned as the safe footprints; otherwise, \( nur \) is inserted into \( st \), and the above steps are repeated.

We use an example to illustrate the main idea. To be clear, we zoom in the cells \((c_x \in \{4, 9\} \) and \( c_y \in \{3, 8\})\) in Fig. 5(a) and show them in Fig. 5(b). Assume that the user \( u \) locates in the cell \((6, 6)\). A 0-rect is found. As the extending rule for a 0-rect (i.e., Rule 0), an \( r \)-rect and an \( l \)-rect are found in the cell \((7, 6)\) and the cell \((5, 6)\) respectively. A \( t \)-rect and a \( b \)-rect are found in the cell \((6, 7)\) and the cell \((6, 5)\) respectively. Then, four borders of a user rectangle are fixed \( ur = p_{1}p_{2}p_{5}p_{4} \). If \( ur \).area < \( \min ur \), \( ur \) is inserted into a stack. Now, the top item in the stack is \( p_{1}p_{2}p_{5}p_{4} \). As an opening boundary selection rule, \( p_{3}p_{4} \) is assumed to be opened. It means \( u_{1} \) is removed, which results in \( p_{2}p_{3} \) being opened as well. With \( u_{1} \) being removed, \( ur \) changes to a special \( bl \)-rect, which traverses more than one cell. As the extending rule at opening points, a new user rectangle \( nur = p_{1}r_{p}p_{6}p_{5} \) is got. Assume \( nur \).area > \( \min nur \), \( u_{2} \) and \( u_{3} \) Related to \( p_{1}p_{7}p_{6}p_{5} \) are returned as the footprints. LCF\(^2\) is shown in Algorithm 2.

**Algorithm 2 LCF\(^2\): Local Cell Footprints Finding Algorithm**

1. locate the tightest \( m \)-rect \( rc \) using \( u \)'s location \((x, y)\);
2. \( rc \) is extended to \( ur \) by applying extending rules;
3. insert \( ur \) into a stack \( st \);
4. while \( st \) is not empty do
5. \( ur\)=get the top item from \( st \);
6. if \( ur \).area > \( \min ur \) then
7. return users related to \( ur \) as the footprints;
8. if all four borders have been opened then
9. pop the top item from \( st \);
10. else
11. open the boundary that hasn’t been opened;
12. extend the opened rectangle to a new user rectangle \( ur \);
13. insert \( ur \) into \( st \);
a user rectangle is saved in a stack? (3) What are the extending rules for finding a user rectangle? (4) If minu is not satisfied, which boundary is selected to be opened?

1) The Tightest m-rect and The Stack: Since more users are expected to be returned as the safe footprints, we define the priority order of m-rect as 2-rect > 1-rect > 0-rect. When a user locates in more than one m-rects, a m-rect with the highest priority is selected.

A user rectangle ur is inserted into a stack if ur.area < minu. The stack is in a decreasing order with the number of users related to ur as the key. Each item ur in the stack includes: (1) the bottom-left and top-right coordinates of ur; (2) users related to ur; (3) a bitstring representing whether the left, right, bottom or top boundary has been opened. When four boundaries have been opened, ur is popped from the stack.

2) Extending Rules: With the aim of locating new boundaries, the extending rules are used in two cases: one is to find a user rectangle ur for a m-rect; the other is to find a new user rectangle ur when some borders are opened. The extending rules in the second case can be revised from the ones in the first case. Thus, we show the extending rules of m-rects first.

Let rc be the tightest m-rect where a user locates. The basic idea of extending is to find the corresponding types of m-rects to locate a new left, right, bottom or top user boundaries in neighbor cells. Due to Theorem 3, not every m-rect in a cell needs to be checked. We only consider the proper m-rects. Taking 0-rects as an example, to locate a right bound, r-rects, br-rects, and tr-rects are supposed to be checked. However, in our method, only r-rects are used to locate the new right bound (details see Rule 0). That is because neither br-rects nor tr-rects will occur before an r-rect is encountered. This point is assured by Theorem 3, which can be proved by contradiction.

Rule 0: (0-rects) Let the cell where the user locates be $(c_{x_0}, c_{y_0})$. Starting from $p = c_{x_0} + 1$ ($p = c_{x_0} - 1$), the first r-rect (l-rect) found in the cell $(p, c_{y_0})$ determines the right (left) border of ur when $p$ increases (decreases) by 1. Symmetrically, starting from $p = c_{y_0} + 1$ ($p = c_{y_0} - 1$), the first t-rect (b-rect) found in the cell $(c_{x_0}, p)$ determines the top (bottom) border of ur when $p$ increases (decreases) by 1.

For an m-rect (m=1 or 2), m borders of ur have been fixed by the m-rect itself. For instance, the right of ur inherits the right of rc for r-rects (m=1). Thus, the rules for m-rects (m=1 or 2) are used to find the other 4-m unknown borders. The searching idea is similar, which can be revised easily from Rule 0. For space limit, the specific rules for m-rects (m=1 or 2) are omitted.

When a boundary of ur is opened, a new boundary is expected to be found on the opened direction. If more than one border is opened, a ur becomes a special m-rect. The corresponding extending rules for m-rects are applied, but the searching start cell changes to the ones where the opened boundaries located. If only one border is opened, related m-rects are checked on the opened direction. For example, if the right is opened, the first tr-rect, br-rect or r-rect encountered determines the new right. The start cell scanned is the cell where the old right locates.

3) Removing Users Selection: Recall FBuck has three options for removing users selection. However, Option 2 and Option 3 in FBuck cannot be applied in LCF², since LCF² has no global PRs information. In addition, FBuck removes users one by one. In order to improve the efficiency, we propose three strategies for users removing on the basis of the boundary. Each time a boundary is opened, all users on the boundary are removed together. We revise Option 1 in FBuck as follows on the basis of the boundary.

Option 1: Four borders are opened as the increasing border sequence. The border score is defined based on the inner distance and the number of boundary users.

Definition 10. (Score) Let NUMty be the number of ty boundary users of ur, where $ty \in \{l, r, b, t\}$. The border score is defined as $score_{ty} = indist_{ty} \times NUMty$.

Option 1 aims to find a safe user rectangle earlier. Option 2 intends to obtain a large footprints set. However, it is unknown when minu will be satisfied, and how many users will remain when different boundaries are opened. Thus, we estimate the number of remaining users in an safe obfuscated location with different boundaries being opened. $d = \frac{\text{min} - \text{AREA}(ur)}{\text{MIN}(\text{width}, \text{height})}$ is the farthest distance by which ur is extended to achieve minu. The number of remaining users $RM_{dy}$, who locate at $d$ far away from ur in the ty direction, is counted, where $ty \in \{l, r, b, t\}$. If $RM_{dy}$ in any direction is 0, $d$ is divided by 2 until $RM_{dy} \neq 0$. Based on this idea, we propose Option 2.

Option 2: The larger $RM_{dy}$ is, the sooner ty is selected to be opened.

Option 3: The boundaries are opened in a random order, but the horizontal and vertical directions are selected alternately. For example, a random opening order is $\{r, b, l, t\}$.

VIII. PERFORMANCE EVALUATION

In this section, the effectiveness and efficiency of our proposed algorithms, FBuck and LCF², are experimentally evaluated under different system settings. The evaluation metrics include the downloaded PRs size, query time, location protection cost at the client’s site, and index updating time at the server site. None of the existing algorithms considers a social discovery service with posterior privacy screening. Hence, we only compare FBuck and LCF² with the baseline. The algorithms are implemented in C++ and evaluated on a desktop running Windows 7 with an Intel Core 2.1GHz CPU and 4GB main memory.

To the best of our knowledge, no real large-scale social discovery datasets have been publically released to date. Therefore, we use a real check-in data crawled from Foursquare [5], which has more than 20 million users and 2 billion check-ins records. We extract the most dense 64*64 km² subspace as a service space. Real venues simulate seeking users, and real check-in users are discovered users. A total of 20,238 simulated users are in the space. The cell size of the grid is set to 0.3% of the space width. By default, the proximity range $R$ for each seeking user is 4% of the space width. For each
discovered user, the width of location uncertain requirement is 200 m.

A. Different Options for Removing User Selection

Different selections on removing users affect the query time and the location protection cost. Query time is the time to find safe footprints at the client site. Location protection cost is how much QoS is sacrificed to protect privacy. The cost is evaluated by \(1-(FS_u(U_{pt}))\), where \(FS_u(U_{pt})\) is the footprints set with (without) privacy protection. Based on the experimental results, a default option for removing users selection is set for LCF\(^2\) and FBuck in the following experiments.

For LCF\(^2\), Option 3 shows the best query time in Fig. 6(a). Option 1 is slighter higher than Option 3, whereas Option 2 is the worst. That is because more time is spent by Option 2 on finding the direction with the most remaining seeking users iteratively. In contrast, the location protection cost using Option 2 is minimum among the three options (see Fig. 6(b)). Though the location protection cost for Option 3 is higher than Option 2, less than 2% redundant cost is sacrificed. Considering the tradeoff between short query time and low query cost, we apply Option 3, a random selection, as the default setting for LCF\(^2\).

For FBuck, we observe that Option 3 shows the best performance on both query time and location protection cost. That is because Option 3 considers both inner distances and outer distances. Thus, Option 3 is used as the default setting. We also observe that the performance using Option 2 and Option 3 looks similar. That implies the effect of outer distances dominates inner distances. Comparing the performances of Option 1 between FBuck and LCF\(^2\), we can see that the query time of LCF\(^2\) is better than FBuck, since several users are removed when a boundary is opened. In contrast, the location protection cost of LCF\(^2\) is higher than FBuck.

B. Different Minimum Uncertain Requirements

Since Baseline and FBuck both use the HGrid, Baseline is not included in evaluation on size of downloaded PRs and index updating time. The PRs downloaded size indicates the communication cost between mobile devices and SPs. Recall that LCF\(^2\) downloads a partial PRs. In our experiments, we evaluate the downloaded size in the densest sub-space. The size of the sub-space is 100 times to the most constraint uncertain requirement. Fig. 7(a) shows the effect of \(\text{min}_u\) on the size of downloaded PRs. The downloaded PRs size doesn’t change for the two methods. Since the global PRs are downloaded, the downloaded size of FBuck is about two times larger than LCF\(^2\). More sophisticated pruning techniques can be done on the SP site to reduce the download size, which due to space limitation is beyond the scope of this paper.

Fig. 7(b) shows the trend of query time at various \(\text{min}_u\) settings. The actual query time of Baseline is too worse to be shown. In order to improve the efficiency of Baseline, we apply an approximate computation that an anchor is used to represent the users with a group of close PRs. The query time of three methods increases with \(\text{min}_u\) since a user requires a large obfuscated location. Among three algorithms, Baseline shows the worst query time even though an approximate computation is employed. When \(\text{min}_u\) is small (less than 150\(^2\)), FBuck is faster than LCF\(^2\), since LCF\(^2\) spends more time on searching proper \(m\)-rects in neighbor cells. When \(\text{min}_u\) continues to increase, LCF\(^2\) becomes the best one. The reason is that, when \(\text{min}_u\) becomes large, more users are removed to enlarge the intersection area. FBuck iterates to delete the proper users one by one. By contrast, a boundary is opened by LCF\(^2\), which removes several users once.

Fig. 7(c) shows that the location protection costs of three algorithms increase with \(\text{min}_u\) increasing. Since Baseline finds a maximum footprints set greedily, the location protection cost of Baseline is smallest among three algorithms. Global PRs is used in FBuck. Thus, the location protection cost of FBuck is smaller than LCF\(^2\) at most cases. When \(\text{min}_u\) increases to 300\(^2\), LCF\(^2\) outperforms FBuck. Generally, global PRs information is helpful to find a safe footprints set with small privacy protection cost.

The index updating time are the insertion time and deletion time at the SP, when a user with PR is inserted into or deleted from the index. For GDGrid, the insertion time dominates the deletion time. Recall in a DGrid, certain user’s PRs are shifted to the cell boundary due to Theorem 3. As a result, an insertion may lead to several users deletions and re-insertions. Thus, we utilize the average insertion time to show the change trend of the index updating time. Fig. 7(d) shows increasing \(\text{min}_u\) has no effect on the insertion time. As expected, the insertion time of HGrid is much smaller than DGrid. Even so, the average insertion time of DGrid is about 0.06s, which is acceptable for SPs.

C. Different Proximity Ranges

In this section, the range \((R)\) of PR varies in [1%, 4%] of the space width. Fig. 8(a) shows the changes of downloaded PRs size. In general, the downloaded size increases with \(R\). Increasing \(R\) indicates more PRs intersect with each other, and more information needs to be saved in an index. Specifically, an enlarged PR covers more cells, thus the grid size increases for FBuck; the number of \(m\)-rects in a cell also increases for LCF\(^2\). When \(R\) is 1% of the space width, the downloaded size of FBuck and LCF\(^2\) is similar. When \(R\) continues to enlarge, the advantage of partly downloading is obvious. LCF\(^2\) outperforms FBuck. Fig. 8(b) shows the query time of three algorithms increases with \(R\) enlarging, since a user locates in more seeking user’s PRs. Since Baseline is a greedy algorithm, its query time grows exponentially when \(R\) increases. Though
the query time of FBuck and LCF also increases, their rise rates are very small. When \( R \) increases to 4%, the query time of FBuck is 0.08s, while LCF needs 0.03s.

From Fig. 8(c), we observe that the location protection cost of three algorithms decreases with the increase of \( R \). That is because the intersection area of PRs becomes large on average. Then, fewer users are removed from a candidate users set to satisfy the personalized uncertain requirements. For location protection cost, FBuck and Baseline outperform LCF at all settings. When \( R \) is small, the cost gap between LCF and FBuck (Baseline) is obvious. When \( R \) is large, the gap shrinks. Similar to Fig. 7(d), the insertion time of HGrid is better than DGrid. From Fig. 8(d), enlarging \( R \) has little effect on the insertion time for HGrid. The insertion time of DGrid increases sharply. The number of \( m \)-rects in a cell increases with \( R \). As a result, DGrid takes more time on affected users deletions and re-insertions.

IX. CONCLUSION

In this paper, we investigated two privacy enhanced social discovery algorithms with posterior screening. We observed that traditional social discoveries cannot provide secure services in standard security model. To address this problem, we propose a novel framework DistSD, where a discovered user's location privacy is ensured through posterior screening in mobile devices. Two heuristic algorithms (FBuck and LCF) are proposed under the new framework. Experimental results show that FBuck has the best efficiency at the server side, but the downloaded size is high. LCF has a small downloaded size and the best query efficiency at client site, whereas the location protection cost is a little high but acceptable. Considering a limited network bandwidth, LCF achieves a good tradeoff between the privacy protection and the query cost.

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